Jurnal Akuntansi dan Keuangan, Vol. 21, No. 1, May 2019, 12-20
DOI: 10.9744/jak.21.1.12-20
ISSN 1411-0288 print / ISSN 2338-8137 online

Modeling of Returns Volatility using GARCH (1,1) Model under Tukey Tranformations

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ABSTRACT

This study proposed two new classes of GARCH (1,1) model by applying the Tukey transformations to the returns and to the lagged variance. The behavior of return volatility was investigated on the basis of models with normal and Student-*t* distributions for return error. The competing models were estimated by using the Excel Solver and Matlab tools. The empirical analysis is based on simulated data, daily exchange rates of the IDR/USD, and daily stock indices of FTSE100 and TOPIX. This study recommends the use of Excel Solver for finance academics and practitioners working on volatility using GARCH (1,1) models. Our empirical findings conclude that GARCH (1,1) models under Tukey transformations should be considered in risk management decisions since the models are more appropriate than standard for describing returns and volatility of financial time series and its stylized facts including fat tails and mean reverting. The Tukey transformed returns imply a shorter volatility half-life, and thus this study suggests that investors should invest the observed assets in a shorter time period to obtain higher returns.

Keywords: Tukey transformation; excel Solver; GARCH; matlab; volatility; JEL classification; C22; C51; C58.

INTRODUCTION

Volatility of asset returns has an essential role to play in all markets because the financial volatility can be statistically interpreted as the standard deviation of the returns changes in the specific time period. It is well known that the financial volatility is typically heteroscedastic, which means the volatility changes over time. A popular class for the time-varying volatility among practitioners in the finance area is the GARCH model proposed by [7], which is a generalization of the ARCH model proposed by [16].

Several GARCH extensions have been introduced in the finance literature to improve some aspects of the GARCH model so that the models are more flexible and adequate in accommodating some characteristics and dynamics of a time series, for example, see [9] and [22] for the survey of GARCH-type models. One extension to the GARCH model is to apply the Box–Cox (BC) transformations for the conditional variance (squared volatility) specification as introduced by [22] in the context of ARCH model and by [21] in the context of GARCH model, where the transformed variance follows a pure autoregressive process. In the context of Stochastic Volatility (SV) models, BC transformation was applied in the lagged variance of the log-variance process by [32] and [25] and in the Realized Variance (RV) data by [26]. Empirically, they showed that the proposed model is superior than the standard model. Alternatively, [31] and [35] applied the Box–Cox transformations to the asset returns under the GARCH and ARCH specifications, respectively. Sarkar provides a maximum likelihood method and a Lagrange multiplier test and then empirically shows that the extended Box–Cox transformation is strongly favored. Meanwhile, [35] apply the shifted BC Box–Cox transformation and provide a second order least square method to estimate the proposed model.

Motivated by the previous studies, the main contribution of this study is to extend the GARCH model by applying the simple power transformations family. First, this study modifies and extends the models proposed in [31] and [35] to the GARCH specification with Student-*t* distributed errors. Second, this study applies the simple power transformation in the lagged variance of the GARCH process. Third, this study applies two proposed models by employing normal and Student-*t* distributions for return error. Many empirical studies, e.g. see [8,36,12], and [11], showed that majority of the asset return errors are not normally distributed. Furthermore, the empirical studies reveal a fact that the financial return distributions are leptokurtic, i.e. the returns have heavy/fat tails. Therefore, the assumption of the Student-t distribution would be much appropriate than the normal distribution assumption. The performance of both extensions is investigated on the GARCH (1,1) model adopting the simulated and real data. Fourth, this study investigates the use of Excel Solver to estimate the extension models. The parameter estimates implied by Excel Solver are compared with those obtained using Markov Chain Monte Carlo (MCMC) procedure in the Matlab software. To the best authors knowledge, the contribution of our study is the first in literature.

This paper is organized as follows. Section 2 reviews the past literature of GARCH modeling and summarizes the relevant articles. Section 3 describes with the proposed models and estimators, the data used in this study, the criteria for adequacy of models, and some properties of the parameters. Empirical results based on simulation data and application for the daily exchange rate and stock data is reported in Section 4. Finally, Section 5 gives some conclusions and extensions for the future work.

The GARCH (1,1) model is perhaps the most popular model in the GARCH-type models and is often used in many empirical studies in the field of finance. The model implies that today's variance can be predicted based on yesterday's squared residuals and variance. [19] compared 330 ARCH-type models and they found that there is no empirical evidence that the GARCH (1,1) model is outperformed by other models. The GARCH (1,1) modeling framework is expressed

$$\left.\begin{aligned} R_t &= \mu_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_t^2) \\ {\sigma_t}^2 &= \omega + \alpha R_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned}\right\} \qquad (1)$$

where $\omega > 0$, $\alpha \geq 0$, and $\beta \geq 0$ to assure positive conditional variances ${\sigma_t}^2$, and $0 \leq \alpha + \beta < 1$ ensures variance stationarity. In the model (1), μ_t is the mean of the return and ε_t is the innovation (also called shock, error or mean residual). For daily data it can often assume that $\mu_t = 0$ as in this study.

The GARCH (1,1)-type models was applied by [20] to Indonesian commodity market, [28] to Indonesian foreign exchange market, [5] to Indonesian stock market, and [15] to Indonesian capital market. [20] examined the predictability of five GARCH-type models, namely ARCH, GARCH, GARCH-M, EGARCH, and TGARCH, for seven primary agricultural commodities in Indonesian export and found that the predictability of the considered models is different for each commodity. [28] applied the GARCH (1,1) model and some of its variations, such as ARCH(1), TARCH(1,1), TS-GARCH (1,1), GJR-GARCH (1,1), NARCH(1), and APARCH(1,1), for the daily selling exchange rates of the EUR (Euro), JPY (Japanese Yen), and USD (US Dollar) against the IDR (Indonesian Rupiah) covering period from January 2010 to December 2015 and found that the GARCH (1,1) model provided the best fit for the selling rates EUR data. In that case, they used the Adaptive Random Walk Metropolis (ARWM) method in the MCMC algorithm to estimate the models. Meanwhile, [5] applied the GARCH (1,1) model to the daily data prices of seven Indonesian stocks for the period from July 2007 to September 2015. They estimate the model using the maximum likelihood estimation method. The results show that GARCH (1,1) model provides evidence of volatility clustering for returns from the prices of Indonesian stocks. [15] compared the volatility shock persistence sectoral indexes between the sectors of consumer goods (CONS) and property-real estate (PROP) for the period from January 2010 to December 2015. Due to the volatility shock of both indices moves back to normal stability quite quickly, [15] recommended to the investors who avoid the risk to invest in both sectors.

In the previous section this study mentioned that financial returns data in fact are not normally distributed, so most often the return error is assumed to follow the Student-t distribution. This assumption was applied by [27] to the APARCH (1,1) model which adopts the exchange rates of five foreign currencies against the IDR for the daily period from January 2010 to December 2016. Their empirical results show that the model with the Student-t distribution provides a better fitting compared to model with the normal distribution.

Alternatively, there are several functions that transform a non-normal distribution into a normal distribution or approximately so. One of these methods that is commonly used both in theoretical work and in practical applications is the Box–Cox (BC) transformation proposed by [10]. Since the BC transformation only works with positive values, they also proposed a modification form of the BC transformation, called the shifted BC (SBC) transformation, which applicables to data containing negative values. Another modification which incorporates unbounded support for the tranformed data was suggested by [6], known as extended BC (EBC) transformations.

The EBC and SBC transformations, respectively, were applied by [31] and [35] to transform the return data with its volatility following GARCH-type models. [31] employed the maximum likelihood method to estimate GARCH model and also developed a Lagrange Multiplier test to check the model adequacy. The model was examined on the daily closing prices of the Bombay Stock Exchange Sensitive Index covering the period from January 1984 to January 1996. The empirical findings show that the proposed model is strongly favored. Meanwhile, [35] proposed the second order least square method to estimate the ARCH(1,1) model.

In the context of an extension of the volatility equation in asymmetric GARCH models, the BC transformation was applied by [21] to both response and regressor. Meanwhile, in the context of stochastic volatility models, [32] and [25] applied BC transformations to lagged volatility. Their empirical results show that the proposed models provide better performance than the standard model.

RESEARCH METHOD

Extensions of GARCH model

A lot of statistical analysis requires the normality assumption for variable. When a variable is not normally distributed, there is a family of power transformations, such as the BC power transformations, may help to normalizing variable. [10] proposed a family of functions that can transform non-normal variable into approximately normal variable. The form of the BC transformation of the variable x is given by:

$$f(x,\lambda) = \begin{cases} \dfrac{x^\lambda - 1}{\lambda} & ,\lambda \neq 0, \\ \log x & ,\lambda = 0, \end{cases}$$

where$x > 0$. This transformation was modified by [6] to the EBC transformation so that the transformation applies for each values of a variable when > 0:

$$f(x,\lambda) = \frac{|x|^\lambda \text{sign}(x) - 1}{\lambda}.$$

Nevertheles, due to the fact that the variance of variable does not change by linear transforming, this study considers the following alternative version proposed by [33], called Extended Simple Tukey (EST) transformation:

$f_{EST}(x,\lambda) = |x|^\lambda \text{sign}(x), \lambda > 0.$

Furthermore, following [31], we define

$f_{EST}(x,\lambda) = \text{sign}(x) \log |x|, \lambda > 0.$

Although not reported, the early empirical studies as in [31] showed that the Sarkar's model is outperformed by the standard model in all observed data cases.

This study now proposes the first generalization of the GARCH (1,1) given as follows:

$$\left.\begin{aligned} f_{EST}(R_t,\lambda_1) = \varepsilon_t, \quad \varepsilon_t \sim N(0,\sigma_t^2) \\ {\sigma_t}^2 = \omega + \alpha R_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned}\right\} \qquad (2)$$

where $f_{EST}(R_t,\lambda_1)$ follows the EST transformations. The above model is then called the ESTR-GARCH (1,1) model. The second generalization is to use the Simple Tukey (ST) transformation to transform the lagged variance. The extension model called ST(1)-GARCH (1,1) is given by :

$$\left.\begin{aligned} R_t = \varepsilon_t, \quad \varepsilon_t \sim N(0,\sigma_t^2) \\ {\sigma_t}^2 = \omega + \alpha R_{t-1}^2 + \beta f_{ST}(\sigma_t^2,\lambda_2) \end{aligned}\right\} \qquad (3)$$

where

$$f_{ST}(\sigma_t^2,\lambda_2) = \begin{cases} (\sigma_t^2)^{\lambda_2} & ,\lambda_2 \neq 0, \\ \log(\sigma_t^2) & ,\lambda_2 = 0. \end{cases}$$

Notice that the value of $\lambda_i = 1$, for i = 1, 2, corresponds to no transformation.

Distribution of Return Error

The extended GARCH (1,1) models are then estimated on the basis of the log-likelihood function. When ε_t in Eqs. (2) and (3) is assumed to follow a normal distribution with mean zero and standard deviation σ_t, the likelihood of the original data consists of the likelihood of the transformed data multiplied by the absolute value of the Jacobian of EST transformation. Thus, the log-likelihood (LL) function of models is given by:

$$\begin{aligned} \log L(\omega,\alpha,\beta,\lambda_1,\lambda_2|R_1,R_2,\ldots,R_T) \\ = -\frac{1}{2}\sum_{t=1}^{T}\left[\log(2\pi\sigma_t^2) + \frac{f_{EST}^2(R_t,\lambda_1)}{{\sigma_t}^2}\right] \\ + \left(\sum_{t=1}^{T}\log|J(R_t,\lambda_1)|\right), \end{aligned}$$

where the process of conditional variance σ_t^2 follows the ST transformation. Following [8], when ε_t is conditionally Student-*t* distributed with mean zero, standard deviation σ_t, and degrees of freedom ν, the LL function of model takes the form

$$\begin{aligned} \log L(\omega,\alpha,\beta,\lambda_1,\lambda_2|R_1,R_2,\ldots,R_T) \\ = \log\Gamma\left(\frac{\nu+1}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right) - \ln\Gamma\left(\frac{1}{2}\right) \\ - \frac{1}{2}\log\left(\sigma_t^2(\nu-2)\right) \\ - \frac{\nu+1}{2}\log\left(1 + \frac{f_{EST}^2(R_t,\lambda_1)}{\sigma_t^2(\nu-2)}\right) \\ + \left(\sum_{t=1}^{T}\log|J(R_t,\lambda_1)|\right), \end{aligned}$$

where σ_t^2 follows the ST transformation. The Student-*t* distribution is a bell-shaped, symmetric, and centered around zero, like the normal distribution, but has heavier tails than the normal. As the number of the degrees of freedom increases, the Student-*t* distribution converge to the standard normal distribution [3].

Estimation Tools

This study uses Excel Solver and Matlab to estimate the model parameters that maximize the log-likelihood function. Excel Solver is preferred by the financial practitioners who do not have strong programming knowledge. The use of Excel Solver to estimate the standard GARCH (1,1) model was studied by, e.g. [2,34], and [29]. In particular, the steps involved in estimating the considered models follow [29].

To analyse whether the use of Excel Solver is recommended in practice, the estimation results are confirmed using both simulation and limited real data. In the simulation case, the accuracy of Excel Solver estimates compared to the true values is measured by relative error. In the case of real data, estimates obtained by the GRG Non-Linear method in Excel Solver and by the Adaptive Random Walk Metropolis (ARWM) method in Matlab are compared. The latter method was proposed by [4] and employed by [24] for the standard GARCH (1,1) model. They found that the method to be statistically efficient and computationally fast.

Data

An empirical comparison of competing models is investigated by fitting them to the daily exchange rate returns of the IDR against the US Dollar (USD) and the daily international stock returns including FTSE100 (Financial Times Stock Exchange 100 Index) and TOPIX (Tokyo Stock Price Index). Notice that the USD is one of the biggest currencies in the world's economy [30]. This currency is the most traded currency in the forex market and the world's leading reserve currency. The FTSE100 Index is a share index of the 100 top-performing companies traded on the London Stock Exchange with the highest market capitalisation [37]. This index is the most frequently used indicator for the UK stock market. Meanwhile, TOPIX is the most important index for the Tokyo Stock Exchange (TSE) in Japan, tracking all domestic companies listed on the TSE's First Section [23]. The TSE is the fourth largest stock exchange in the world.

The daily closing prices of buying rates IDR/USD are obtained from the webpage of Bank Indonesia (www.bi.go.id) over the period from January 2010 to December 2017, excluding weekends and national holidays. The daily returns of FTSE100 cover data from January 2000 to December 2013, which are publicly available from Oxford-Man Institute's Realised Library (https://realized.oxford-man.ox.ac.uk/data/download), while the daily returns of TOPIX cover data from January 2000 to December 2014 and are available from the corresponding author upon request.

Table 1. Descriptive statistics.

Stats.	IDR/USD	FTSE100	TOPIX
Observations	1891	3509	3665
Mean	0.0197	–0.0385	–0.0073
Median	0.0314	–0.0061	0.0274
Std. Dev.	0.4378	0.9979	1.4138
Maximum	2.7128	7.0441	12.8646
Minimum	–2.8583	–5.7603	–10.0071
Skewness	–0.3501	–0.1399	–0.3438
Kurtosis	9.17	6.98	8.93
JB stats.	3036.7	2321.5	5434.9
Crit. Val.	5.96	5.98	5.98

The return series of exchange rates were calculated by 100 times the logarithmic difference of the closing price of the current day and the closing price of the previous day, i.e.

$$R_t = 100 \times [\ln(P_t) - \ln(P_{t-1})],$$

where P_t is the exchange rate at day t. Table 1 presents the descriptive statistics for the daily returns. The daily mean value for all returns is close to zero as the previous assumption. The largest standard deviation and widest range are given by TOPIX, which indicate that the fluctuation of TOPIX returns is most significant. Since the values of skewness and kurtosis indicate a distribution type of the data, it is important to test it on the observed data. Skewness values near zero, indicating the distribution of all returns is relatively symmetric around its mean value. The kurtosis value significantly exceeds 3 for all assets, which shows evidence of heavy tails. Therefore, it can be assumed that the distribution of returns in all assets is symmetric and has a heavier tail than the normal distribution. The Jarque–Bera (JB) test confirms that the distribution of asset returns is not normally distributed, indicated by the value of JB statistic is greater than the critical value.

Evaluation of Model

To select the appropriate model between M_0 and M_1 which provides the best fitting model, this study uses discrimination criteria such as the Log-likelihood Ratio (LLR) statistics:

$$LLR_{M_1.M_0} = 2\left(\log L_{M_1} - \log L_{M_0}\right),$$

where the critical values of χ^2 distribution with 1 degree of freedom at significance levels 1%, 5%, and 10% are 6.64, 3.84, and 2.71, respectively.

This study will also complete the analysis based on some characteristics driven by volatility persistence $\phi = \alpha + \beta$, e.g. see [1,18], dan [38], and, such as unconditional (long-run, average) volatility defined by $V_L = \frac{\omega}{1-\phi}$ and half-life of a volatility shock defined by $L_h = \frac{\log 0.5}{\log(\phi)}$. The half-life of volatility shock measures the time periods (number of days in this study case) for the variance to move half way back towards its unconditional volatility. According to [1], an asset having the half-life of a volatility shock about n days suggests to investors should open a position at 0 days and must close after $2n$ days.

RESULT AND DISCUSSION

This section conducts simulation and empirical studies to compare the performance of the competing models and to investigate the use of Excel Solver.

Simulation Study

The main purpose of this simulation study is to demonstrate: (1) whether Excel Solver is able to estimate the parameter of considered models in terms of relative error, and (2) whether the proposed models outperforms the basic model.

Two return series are generated from ESTR-GARCH (1,1) and ST(1)-GARCH (1,1) models with normal distribution for the return error. Each simulated data set is the same length, i.e. 1000 observations. The true parameter values for each model were set as in Tables 1 and 2 based on the result of empirical studies in financial literatures. Relative error between the true parameter and its estimate was calculated to measure the accuracy of the Excel Solver estimates. Excel Solver was initialized by setting $\omega = 0.030$, $\alpha = 0.040$, $\beta = 0.95$, and $\lambda_1 = 0.85$ with the log-likelihood value of –1420.51 for fitting into the ESTR-GARCH (1,1) model and $\omega = 0.003$, $\alpha = 0.2$, $\beta = 0.75$, and $\lambda_2 = 0.95$ with the log-likelihood value of –848.28 for fitting into the ST(1)-GARCH (1,1) model. Each simulated data set was also fitted into the GARCH (1,1) model to compare their performances.

Tables 2 and 3 summarize the simulation results, including the parameter estimates and log-likelihood value. First, in terms of relative error, all cases demonstrate that Excel Solver is a reliable tool for estimating the parameters α, β, and $\lambda_{1,2}$ but less reliable for estimating the parameters ω. This study notes that Excel Solver is very sensitive to initial value and a good initial value therefore needs to be close enough to the true value. Poor choice of initial values can lead to an ill-behaved estimate and causes convergence problem. Second, by comparing the log-likelihood values of the proposed and standard models, the value of LLR_{EBCR} is 58.58 which is greater than the greatest critical value of χ^2 distribution with 1 degree of freedom. It indicates that the ESTR-GARCH (1,1) model is significantly better than the GARCH (1,1) model. Meanwhile, the value of $LLR_{BC(1)}$ is 2.30 which is not significant at any conventional level. It means that the GARCH (1,1) and ST(1)-GARCH (1,1) models have same performance.

Table 2. Simulated ESTR-GARCH (1,1) model.

Parameter	True value	ESTR-GARCH (1,1)		GARCH (1,1)
		Estimate	Relative error	Estimate
ω	0.04	0.064	59.0%	0.071
α	0.05	0.032	35.6%	0.033
β	0.90	0.895	0.5%	0.897
λ_1	0.80	0.824	3.1%	-
$\alpha+\beta$	0.95	0.927	-	0.931
Total Log(L)		–1388.30		–1417.59

Table 3. Simulated ST(1)-GARCH (1,1) model.

Parameter	True value	ST(1)-GARCH (1,1)		GARCH (1,1)
		Estimate	Relative error	Estimate
ω	0.005	0.0015	70.0%	0.0248
α	0.250	0.2210	11.6%	0.2197
β	0.700	0.7242	3.5%	0.7249
λ_2	0.900	0.9027	0.3%	-
$\alpha+\beta$	0.950	0.9452	-	0.9446
Total Log(L)		–842.58		–843.73

Application to Real Data

Table 4 reports the empirical results of the competing models which are estimated by using the GRG Non-Linear method in Excel Solver and the ARWM method in Matlab. This study labels the GARCH (1,1), ESTR-GARCH (1,1), and ST(1)-GARCH (1,1) models with normally distributed error as Model (1), (2), and (3), respectively, and then called GARCHn(1,1), ESTR-GARCHn(1,1), and ST(1)-GARCHn(1,1) models. It seems that Excel Solver and Matlab give similar estimates for most cases, even though in the case of ESTR-GARCHn(1,1) model Excel Solver produces $\omega = 0$ for the IDR/USD data and $\alpha + \beta = 1$ for the FTSE100 and TOPIX data, which do not satisfy the model constraints. This is due to the unavailability of strict inequality for the constraints in the Excel Solver tool, the estimate of ω is very close to zero, and the estimate of $\alpha + \beta$ is very close to unity. The situation of $\alpha + \beta = 1$ is known as the Integrated GARCH (IGARCH) model, which was proposed by [17]. IGARCH model implies that the unconditional variance of return series is not finite and multiperiod forecasts of variance will trend upwards [13]. Therefore, the next analysis is on the basis of estimation results obtained by the ARWM method in Matlab.

Table 4. Empirical results of GARCHn(1,1) models.

Data	Model	Tool	ω	α	β	λ_1	λ_2	ϕ	Total Log(L)
IDR/USD	(1)	Solver	0.0063	0.2122	0.7807	-	-	0.9929	–780.84
		Matlab	0.0070	0.2128	0.7739	-	-	0.9867	–782.09
	(2)	Solver	0.0066	0.1915	0.7895	0.8005	-	0.9810	–682.79
		Matlab	0.0074	0.2031	0.7757	0.8003*	-	0.9788	–684.67
	(3)	Solver	0.0000	0.2410	0.7038	-	0.9187	0.9448	–771.14
		Matlab	0.0009	0.2477	0.7028	-	0.9248*	0.9505	–773.50
FTSE100	(1)	Solver	0.0053	0.0901	0.9064	-	-	0.9964	–4260.46
		Matlab	0.0061	0.0929	0.9022	-	-	0.9951	–4261.88
	(2)	Solver	0.0052	0.0866	0.9087	0.9174	-	0.9952	–4239.91
		Matlab	0.0056	0.0872	0.9073	0.9173*	-	0.9945	–4241.83
	(3)	Solver	0.0021	0.0883	0.9117	-	0.9931	1.0000	–4259.72
		Matlab	0.0048	0.0903	0.9057	-	0.9971	0.9960	–4261.67
TOPIX	(1)	Solver	0.0451	0.1092	0.8696	-	-	0.9788	–6040.86
		Matlab	0.0457	0.1104	0.8685	-	-	0.9789	–6042.29
	(2)	Solver	0.0398	0.1029	0.8748	0.9235	-	0.9777	–6022.02
		Matlab	0.0433	0.1077	0.8685	0.9237*	-	0.9762	–6023.97
	(3)	Solver	0.0241	0.1078	0.8922	-	0.9830	1.0000	–6037.74
		Matlab	0.0383	0.1109	0.8775	-	0.9886**	0.9884	–6040.87

Note: * (or **): value deviates significantly from 1 in terms of 95% (or 90%) Highest Posterior Density interval (see [14] for details)

Table 5. Empirical results of GARCHt(1,1) models.

Data	Model	Tool	ω	α	β	λ_1	λ_2	ν	ϕ	Total Log(L)
IDR/USD	(4)	Solver	0.0030	0.2037	0.7963	-	-	4.13	1.0000	–583.65
		Matlab	0.0034	0.2049	0.7896	-	-	4.22	0.9945	–585.83
	(5)	Solver	0.0028	0.2023	0.7977	1.0530	-	3.71	1.0000	–581.42
		Matlab	0.0029	0.1946	0.8005	1.0488*	-	3.81	0.9951	–583.42
	(6)	Solver	0.0000	0.2839	0.7161	-	0.9452	3.82	1.0000	–577.45
		Matlab	0.0009	0.2726	0.7141	-	0.9507*	4.01	0.9867	–580.41
FTSE100	(4)	Solver	0.0046	0.0839	0.9131	-	-	11.33	0.9971	–4238.95
		Matlab	0.0052	0.0833	0.9123	-	-	12.43	0.9956	–4240.97
	(5)	Solver	0.0048	0.0842	0.9119	0.9510	-	19.53	0.9960	–4235.47
		Matlab	0.0058	0.0876	0.9068	0.9484*	-	23.55	0.9944	–4235.93
	(6)	Solver	0.0019	0.0823	0.9177	-	0.9942	11.49	1.0000	–4238.42
		Matlab	0.0048	0.0852	0.9104	-	0.9982	12.71	0.9956	–4241.88
TOPIX	(4)	Solver	0.0379	0.0943	0.8866	-	-	11.51	0.9810	–6007.59
		Matlab	0.0435	0.1004	0.8781	-	-	12.25	0.9785	–6009.84
	(5)	Solver	0.0376	0.0945	0.8860	0.9826	-	13.15	0.9805	–6007.10
		Matlab	0.0522	0.1110	0.8613	0.9696*	-	16.76	0.9723	–6013.34
	(6)	Solver	0.0190	0.0933	0.9067	-	0.9849	11.77	1.0000	–6005.00
		Matlab	0.0262	0.0917	0.9014	-	0.9882*	12.50	0.9931	–6007.78

First, the 95% of HPD (Highest Posterior Density) interval of λ_1 in the ESTR-GARCHn(1,1) model excludes 1 (the basic version) in all cases. Thus, all observed data provide significant evidence to transform the original return data using the EST transformation against no transformation case. In the case of ST(1)-GARCHn(1,1) model, the 95% and 90% of HPD intervals of λ_2 exclude 1 for the IDR/USD and TOPIX data, respectively. Thus, both data provide significant evidence against the basic model. Second, the total log-likelihood estimates and the LLR statistics indicate that the basic GARCHn(1,1) model provide the best fit for all data at any conventional level, whereas the ST(1)-GARCHn(1,1) model provide a better fit than the GARCHn(1,1) for the IDR/USD and TOPIX data. This result is consistent with significance of ST parameter. Both findings indicate the extended GARCHn models have the potential to provide a better fitting than the basic GARCHn model.

Regarding the volatility persistence, the estimate of ϕ in the ESTR-GARCHn(1,1) model is lower than those in the GARCHn(1,1) model, which is suggestive of a low persistency of volatility. It indicates that volatility implied by the ESTR-GARCHn(1,1) model is more volatile (less smooth) but less persistent. With the lower persistence in volatility, the ESTR-GARCHn(1,1) model produces smaller unconditional volatility and shorter volatility half-life than the GARCHn(1,1) model. So, GARCHn(1,1) model under ST transformation for return implies an asset provides a shorter time period to operate freely. It means that the model causes the value of assets to be more sensitive to new information. This recommends that investors of IDR/USD, FTSE100, and TOPIX must open a position at 0 days and must close after the 66th day, 252nd day, and 58th day, respectively. Meanwhile, in the case of ST(1)-GARCH (1,1) model, there is no conclusion about persistence, unconditional volatility, and half-life of volatility. The results of the unconditional volatility and half-life of volatility are presented in Table 6.

Next, this study focuses on the use of the Student-t distribution for the GARCH (1,1), ESTR-GARCH (1,1), and ST(1)-GARCH (1,1) models, respectively then called GARCHt(1,1), ESTR-GARCHt(1,1), and ST(1)-GARCHt(1,1) models, which are labeled as Model (4), (5), and (6) in Table 5. Overall, the empirical results showed that Excel Solver and Matlab provide similar estimation values, except the estimate of degrees of freedom in the ESTR-GARCH (1,1) model adopting the FTSE100 and TOPIX data. In particular, this study found that Solver Excel produces estimates α and β which give $\alpha + \beta = 1$ in the ST(1)-GARCHt(1,1) model for all cases and $\omega = 0$ for the IDR/USD case. This confirms the previous result in the case of normal distribution. However, it can be considered that Solver Excel is quite capable to estimate the GARCH (1,1) models with normal or Student-t distributions for return error.

As the previous discussion, the following analysis is on the basis of estimation results obtained by the ARWM method in Matlab. The *LLR* test indicates that the models with Student-t distribution for returns error provide better fitting than the model with normal distribution. The ESTR-GARCHt(1,1) model provides a better fitting than the GARCHt(1,1) model for IDR/USD and FTSE100 data, indicated by the LLR test statistic of 4.82 and 10.08, respectively, which are significant any 5% level. The ST(1)-GARCHt(1,1) model provides a better fitting than the GARCHt(1,1) model for IDR/USD and TOPIX data, indicated by the LLR test statistics of 10.84 and 4.12, respectively, which are significant at the 5% level. These results indicate that the proposed models have the potential to provide better fitting than the GARCH (1,1) model. In the case of ESTR-GARCH (1,1) model adopting TOPIX data, although the 95% HPD interval of Tukey parameter excludes 1, it does not necessarily confirm the superiority of model.

Table 6. Unconditional volatility V_L and half-life of a volatility shock L_h.

Model	V_L			L_h		
	IDR/USD	FTSE100	TOPIX	IDR/USD	FTSE100	TOPIX
(1)	0.526	1.245	2.166	51.77	141.11	32.50
(2)	0.349	1.018	1.819	32.35	125.68	28.78
(3)	0.018	1.200	3.302	13.65	172.94	59.4
(4)	0.618	1.182	2.023	125.68	157.19	31.89
(5)	0.592	1.036	1.885	141.11	123.43	24.68
(6)	0.068	1.091	3.797	51.77	157.19	100.11

Regarding the degrees of freedom, the models applying ST transformations produce smaller degrees of freedom than the basic model in the IDR/USD case and greater degrees of freedom than the basic model in the FTSE and TOPIX cases. Regarding the parameters ω, α, dan β, Table 6 presents the unconditional volatility and half-life of volatility shock. The unconditional volatility and half life of volatility implied by the the ESTR-GARCHt(1,1) models respectively is smaller and shorter (except the IDR/USD data) than those implied by the GARCHt(1,1). In the case of lagged-volatility transformation, the results of unconditional volatility and half life of volatility is similar in the normal and Student-t cases in terms of comparison with the basic model. The findings indicate that the incorporation of Student-t distribution into the return error does not affect the comparison of unconditional volatility and half life of volatility between the proposed models and the basic model.

Furthermore, the use of Student-t distribution in the ESTR-GARCH (1,1) model increases the unconditional volatility and decreases (except IDR/USD data) the half-life of volatility shock. Meanwhile, the ST(1)-GARCHt(1,1) increases the half-life of volatility shock for IDR/USD and TOPIX data but decreases the half-life of volatility shock for FTSE data. These results indicate that the new shock to volatility of the ESTR-GARCHt(1,1) and ST(1)-GARCHt(1,1) models for financial return tend to affect the returns for shorter and longer periods, respectively. In other words, this suggests that the recent information in the ESTR-GARCHt(1,1) model is more important than old information. In practice, the model provides a shorter time for investors to operate since the asset values are more sensitive to new information.

CONCLUSION

This study investigated the empirical performance of two new non-linear classes of GARCH model by applying the Tukey transformations to the asset returns and the lagged variance. This study assumes that return errors follow normal and Student-t distributions. On the basis of Excel

Solver estimates, this study suggests the use of Excel Solver to estimate the basic GARCH model but the use to estimate the extended GARCH models depends on the observed data. In terms of log-likelihood ratio test, the empirical results showed that the two proposed models have a potential to outperform the basic GARCH model. Furthermore, the results showed a strong evidence incorporating the Student-*t* into the returns error distribution. The results of this study demonstrate that the Tukey transformation provides insight to identify the time period of investment. A transformation for returns provides an opportunity to investors and financial experts for short-term investment the FTSE100 and TOPIX, which has the shortest half-life. In contrast, a transformation for lagged variance suggests that it is better to invest in the FTSE100 and TOPIX for the long-term investment, which has the longest half-life. For the IDR/USD, the short-term investment is recommended by both transformation.

Implication, Limitation, and Suggestion

This study employed the GRG Non-Linear method in Excel Solver and the ARWM method in the Matlab to estimate the considered models. Some estimation results obtained by Excel Solver do not meet the model constraints since Excel Solver does not offer the option "< "nor ">" for constraints. In addition, Excel Solver also does not provide a confidence interval for parameter estimates so that it cannot confirm the significance of the estimates. Therefore, if we are only interested in the parameter estimate value, Excel Solver is enough.

Acknowledgment

The authors gratefully acknowledge research support from: (1) the *Universitas Kristen Satya Wacana, office of the Vice Rector for Research and Public Service, via Mandatory Research 2018–2019, and (2) The Ministry of Technology Research and Higher Education (Kemenristek-Dikti) via Higher Education Excellent Fundamental Research Grant 2019.*

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